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Advanced Linear Algebra (MA 409)
Problem Sheet - 18

Summary – Important Facts about Determinants

1. Label the following statements as true or false.

   (a) The determinant of a square matrix may be computed by expanding the matrix along any row or column.

   (b) In evaluating the determinant of a matrix, it is wise to expand along a row or column containing the largest number of zero entries.

   (c) If two rows or columns of A are identical, then $\det(A) = 0$.

   (d) If B is a matrix obtained by interchanging two rows or two columns of A, then $\det(B) = det(A)$.

   (e) If B is a matrix obtained by multiplying each entry of some row or column of A by a scalar, then $\det(B) = \det(A)$.

   (f) If B is a matrix obtained from A by adding a multiple of some row to a different row, then $\det(B) = \det(A)$.

   (g) The determinant of an upper triangular $n \times n$ matrix is the product of its diagonal entries.

   (h) For every $A \in M_{n\times n}(F)$, $\det(A^t) = -\det(A)$.

   (i) If $A, B \in M_{n\times n}(F)$, then $\det(AB) = \det(A) \cdot \det(B)$.

   (j) If Q is an invertible matrix, then $\det(Q^{-1}) = [\det(Q)]^{-1}$.

   (k) A matrix Q is invertible if and only if $\det(Q) \neq 0$.

2. Evaluate the determinant of the following 2×2 matrices.

   a) $\begin{pmatrix} 4 & -5 \\ 2 & 3 \end{pmatrix}$ b) $\begin{pmatrix} -1 & 7 \\ 3 & 8 \end{pmatrix}$

   c) $\begin{pmatrix} 2+i & -1+3i \\ 1-2i & 3-i \end{pmatrix}$ d) $\begin{pmatrix} 3 & 4i \\ -6i & 2i \end{pmatrix}$

3. Evaluate the determinant of the following matrices in the manner indicated.

   a) $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{pmatrix}$ along the first row

   b) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ -1 & 3 & 0 \end{pmatrix}$ along the first column

c) $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{pmatrix}$ along the second column

d) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ -1 & 3 & 0 \end{pmatrix}$ along the third row

e) $\begin{pmatrix} 0 & 1+i & 2 \\ -2i & 0 & 1-i \\ 3 & 4i & 0 \end{pmatrix}$ along the third row

f) $\begin{pmatrix} i & 2+i & 0 \\ -1 & 3 & 2i \\ 0 & -1 & 1-i \end{pmatrix}$ along the third column

g) $\begin{pmatrix} 0 & 2 & 1 & 3 \\ 1 & 0 & -2 & 2 \\ 3 & -1 & 0 & 1 \\ -1 & 1 & 2 & 0 \end{pmatrix}$ along the fourth column

h) $\begin{pmatrix} 1 & -1 & 2 & -1 \\ -3 & 4 & 1 & -1 \\ 2 & -5 & -3 & 8 \\ -2 & 6 & -4 & 1 \end{pmatrix}$ along the fourth row

4. Evaluate the determinant of the following matrices by any legitimate method.

a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 3 & 2 \\ 4 & -8 & 1 \\ 2 & 2 & 5 \end{pmatrix}$

c) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & -5 \\ 6 & -4 & 3 \end{pmatrix}$

d) $\begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & -5 \\ 3 & -1 & 2 \end{pmatrix}$

e) $\begin{pmatrix} i & 2 & -1 \\ 3 & 1+i & 2 \\ -2i & 1 & 4-i \end{pmatrix}$

f) $\begin{pmatrix} -1 & 2+i & 3 \\ 1-i & i & 1 \\ 3i & 2 & -1+i \end{pmatrix}$

g) $\begin{pmatrix} 1 & 0 & -2 & 3 \\ -3 & 1 & 1 & 2 \\ 0 & 4 & -1 & 1 \\ 2 & 3 & 0 & 1 \end{pmatrix}$

h) $\begin{pmatrix} 1 & -2 & 3 & -12 \\ -5 & 12 & -14 & 19 \\ -9 & 22 & -20 & 31 \\ -4 & 9 & -14 & 15 \end{pmatrix}$

5. Suppose that $M \in M_{n\times n}(F)$ can be written in the form

$$M = \begin{pmatrix} A & B \\ O & I \end{pmatrix},$$

where A is a square matrix. Prove that $\det(M) = \det(A)$.

6. Prove that if $M \in M_{n\times n}(F)$ can be written in the form

$$M = \begin{pmatrix} A & B \\ O & C \end{pmatrix},$$

where A and C are square matrices, then $\det(M) = \det(A) \cdot \det(C)$.

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